

Energy of Graphs with Multiple Self-Loops

M. R. RAJESH KANNA¹ SOWMYA PRABHA B.C² and ROOPA S³

¹Maharani's Science College for Women(Autonomous),
J.L.B. Road, Mysuru-570005, Karnataka.

²Research Scholar,
Sri D. Devaraja Urs Government First Grade College,
(A Recognised Research Centre of the University of Mysore)
Hunsur - 571 105, Karnataka.

³Government College for Women's,
Maddur - 571 428, Karnataka.

(E-mail: mr.rajeshkanna@gmail.com, sowmyaprabha0@gmail.com,
roopa.s.kumar@gmail.com)

Abstract

Building on the foundational work of Professor Ivan Gutman and collaborators, who introduced the concept of graph energy for graphs with self-loops, this study extends the notion to graphs containing multiple self-loops. For a simple graph G with n vertices and m edges, the modified graph G_M is formed by adding r_1, r_2, \dots, r_n self-loops to vertices v_1, v_2, \dots, v_n , respectively.

The energy of G_M is defined as: $E(G_M) = \sum_{i=1}^n \left| \lambda_i(G_M) - \frac{\sigma_M}{n} \right|$

Where $\sigma_M = \sum_{i=1}^n r_i$ represents the total number of self-loops, and $\lambda_1(G_M), \lambda_2(G_M), \dots, \lambda_n(G_M)$ are the eigenvalues of the adjacency matrix of G_M . This work investigates the spectral properties of G_M and derives bounds for its energy, $E(G_M)$. This study focuses on deriving bounds analogous to those of E_G , while highlighting notable differences and identifying opportunities for discovering sharper bounds. The results contribute to advancing the theoretical understanding of spectral graph properties, especially for graphs augmented with multiple self-loops.

Mathematics Subject Classification: 05C12, 05C90.

Keywords and phrases

Graph energy, self-loop, eigenvalue, adjacency matrix, spectral graph theory.